

## Exponential Functions

**Laws of Exponents:** If  $s$ ,  $t$ ,  $a$ , and  $b$  are real numbers with  $a > 0$  and  $b > 0$ ,

$$\text{then } a^s \cdot a^t = a^{s+t} \quad (a^s)^t = a^{st} \quad (ab)^s = a^s b^s \quad 1^s = 1 \quad a^{-s} = \frac{1}{a^s} = \left(\frac{1}{a}\right)^s \quad a^0 = 1$$

An **exponential function** is a function of the form  $f(x) = a^x$ , where  $a$  is a positive real number ( $a > 0$ ) and  $a \neq 1$ . The domain of  $f$  is the set of all real numbers.

**Theorem:** For an exponential function  $f(x) = a^x$ ,  $a > 0$ ,  $a \neq 1$ , if  $x$  is any real number, then

$$\frac{f(x+1)}{f(x)} = a$$

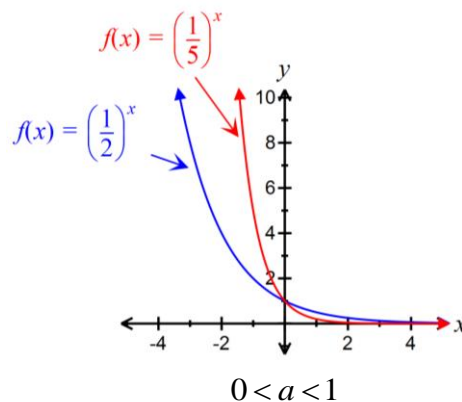
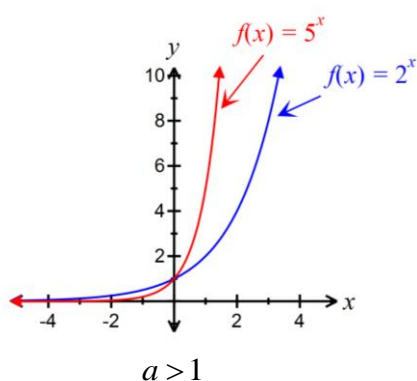
**Examples:** Determine whether the given functions are exponential or not.

$x$	$f(x)$
-1	2
0	5
1	8
2	11
3	14

$x$	$f(x)$
-1	$2/3$
0	1
1	$3/2$
2	$9/4$
3	$27/8$

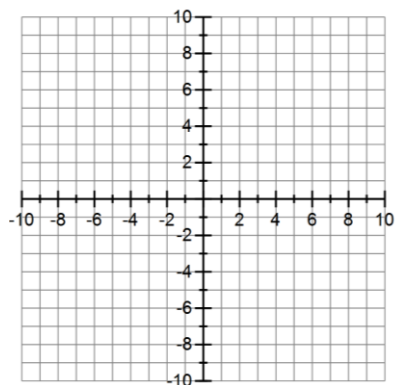
**Properties of the Exponential Function  $f(x) = a^x$ ,  $a > 0$ ,  $a \neq 1$**

- Domain:  $(-\infty, \infty)$ ; Range:  $(0, \infty)$
- There are no  $x$ -intercepts; the  $y$ -intercept is 1.
- The  $x$ -axis ( $y = 0$ ) is a horizontal asymptote.
  - For  $a > 1$ , the graph approaches the  $x$ -axis as  $x \rightarrow -\infty$ .
  - For  $0 < a < 1$ , the graph approaches the  $x$ -axis as  $x \rightarrow \infty$ .
- $f(x) = a^x$  is one-to-one.
  - For  $a > 1$ ,  $f(x) = a^x$  is an increasing function.
  - For  $0 < a < 1$ ,  $f(x) = a^x$  is a decreasing function.
- The graph of  $f$  contains the points  $(-1, \frac{1}{a})$ ,  $(0, 1)$ , and  $(1, a)$ .
- The graph of  $f$  is smooth and continuous, with no corners, gaps, or cusps.

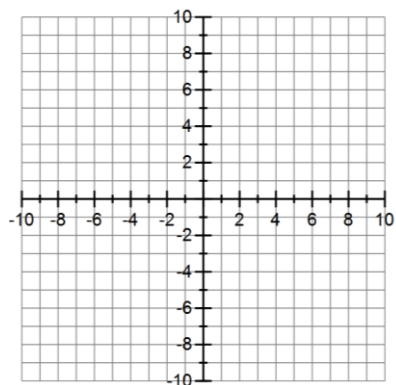


### Examples:

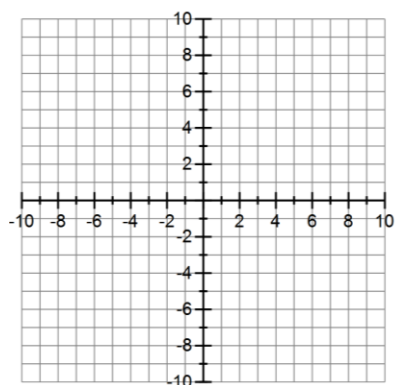
a) Graph  $f(x) = 3^x$ .



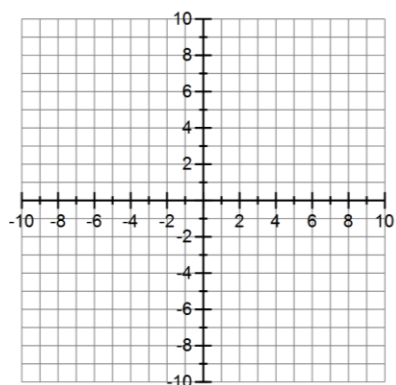
b) Graph  $f(x) = 2 \cdot \left(\frac{1}{3}\right)^x$ .



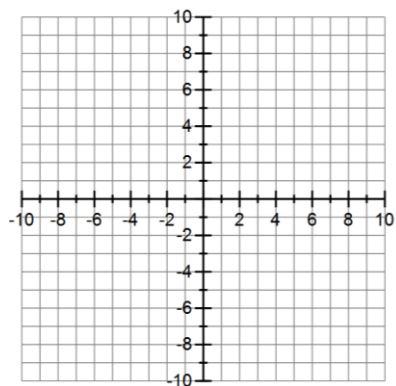
c) Graph  $f(x) = 5^{x+3}$ .



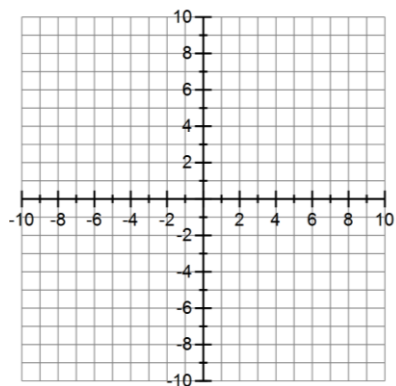
d) Graph  $f(x) = \left(\frac{1}{2}\right)^x + 3$ .



e) Graph  $f(x) = 2^{-x}$ .



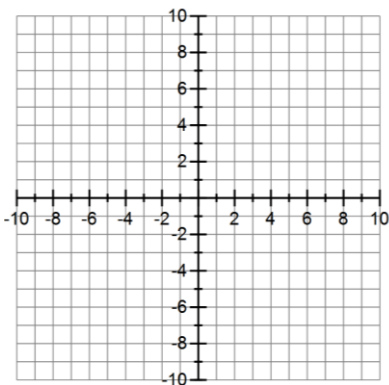
f) Graph  $f(x) = -3^x$ .



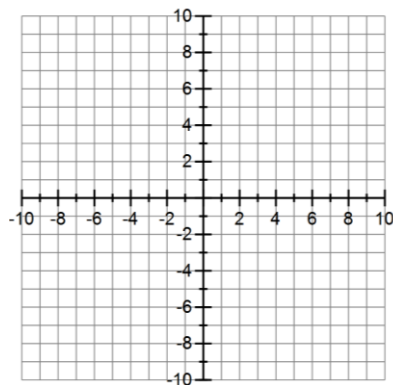
The **number  $e$**  (approximately 2.71828...) is defined as the number that the expression  $\left(1 + \frac{1}{n}\right)^n$  approaches as  $n \rightarrow \infty$ . In calculus, this is expressed using limit notation as  $e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$ .

### Examples:

a) Graph  $f(x) = e^x$ .



b) Graph  $f(x) = -e^{-x}$ .



### Solving Exponential Equations

If  $a > 0$  and  $a \neq 1$  and  $a^u = a^v$ , then  $u = v$ .

Many exponential equations can be rewritten so the two sides have a common base. This allows us to set the exponents equal to each other and solve the equation.

**Examples:** Solve the following equations.

a)  $3^{-x} = 243$

b)  $5^{x+3} = \frac{1}{5}$

c)  $4^{x^2} = 2^x$

d)  $3^{x^2-5x} = \frac{1}{81}$

e)  $4^x \cdot 2^{x^2} = 16^2$

f)  $e^{x^2} = e^{3x} \cdot \frac{1}{e^2}$